SGN-2606 Statistical Signal Processing,
Exam, 15.10.2007

No written material is allowed in the exam. A calculator is allowed, but the memory has to be empty. Write in English or Finnish.

1. Explain briefly (1-2 sentences) the following concepts
   (a) Sufficient statistic
   (b) MMSE estimator
   (c) Efficient estimator
   (d) Linear model
   (e) Likelihood function
   (f) Prior PDF

2. Consider the model
$$x[n] = As[n] + w[n], \qquad n = 0, 1, \ldots, N-1,$$
where $w[n] \sim \mathcal{N}(0, \sigma^2)$, s[n] is a known signal, and A is the parameter to be estimated. Find the MLE of A.

3. N observations {x[0],x[1],...,x[N-1]} are made from the Rayleigh PDF:
$$p(x; \sigma^2) = \begin{cases} \frac{x}{\sigma^2} \exp\left(-\frac{1}{2}\frac{x^2}{\sigma^2}\right), & x \geq 0 \\ 0, & x < 0 \end{cases}$$
Find a method of moments estimator for σ^2. The mean of Rayleigh distribution is
$$\mu = \sigma\sqrt{\frac{\pi}{2}}$$

4. The *Poisson distribution* is a discrete probability distribution that expresses the probability of a number of events $k \geq 0$ occurring in a fixed period of time:
$$p(x; \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$$
The distribution is used to model the number of mutations in a given stretch of DNA after a certain amount of radiation. N stretches are measured to produce a Poisson distributed signal $x[n]$, $n = 0, 1, \ldots, N-1$. Find the maximum likelihood estimator of λ. (4p) Is it unbiased? (2p)

5. The data $x[n] \in \mathbf{R}$ for $n = 0, 1, \ldots, N-1$ are observed, each sample having the conditional PDF
$$p(x[n] \mid \theta) = \begin{cases} \theta \exp(-\theta x[n]), & \text{if } x[n] > 0, \\ 0, & \text{otherwise.} \end{cases}$$
Moreover, suppose that θ is a random variable with prior PDF
$$p(\theta) = \begin{cases} \lambda \exp(-\lambda\theta) & \text{if } \theta > 0, \\ 0, & \text{otherwise.} \end{cases}$$
Find the MAP estimator of θ.