

General instructions:

Use of books and notes allowed.

Provide justified but concise answers.

Write your family name on every non-empty page you hand in,

number consecutively all non-empty pages you hand in,

write the total number of non-empty pages handed in on the first page.

No page should contain answers or fragments of answers to more than one question.

Questions:

1. Make a catalogue of the different finite fields with at most 32 elements, and of the possible containment relationships among these fields.
2. What is the number of those elements of $GF(128)$ that are not contained in any proper subfield? (I.e. not contained in any subfield of $GF(128)$ other than $GF(128)$ itself.)
3. (a) Give a list of all irreducible polynomials of degree three over $GF(3)$.
(b) For each element of $GF(4)$, find its minimal polynomial with respect to the two-element subfield.
4. (a) Can the multiplicative group of an infinite field have a non-trivial finite subgroup?
(b) Can the field of real numbers have a finite subfield?
5. What is the relationship between infinite rings and finite fields?