TLT-5906 Advanced Course in Digital Transmission
Digitaalisen siirtotekniikan jatkokurssi

Exam 20.3. 2006

You can answer in English, Finnish, or Swedish.

1. Sketch the *equivalent discrete-time white noise filter model* for a linear digital transmission system (e.g. PAM, QAM). How (in principle) can the filter model be determined from the transmitted signal pulse $g(t)$ and channel impulse response $c(t)$? Explain the roles of the matched filter and noise whitening filter in the ideal receiver front-end structure. What are the characteristics (power spectrum, autocorrelation) of the AWGN channel noise (a) after the matched filter and (b) at the output of the equivalent discrete-time white noise filter model?

2. A binary communication system transmits the same information on two diversity channels. The two received signals are

$$r_1 = \pm\sqrt{E_b} + n_1$$
$$r_2 = \pm\sqrt{E_b} + n_2$$

where $E(n_1) = E(n_2) = 0$, $E(n_1^2) = \sigma_1^2$ and $E(n_2^2) = \sigma_2^2$, and n_1 and n_2 are uncorrelated Gaussian variables. The detector bases its decision on the linear combination of r_1 and r_2, i.e.,

$r = r_1 + kr_2$

(a) Determine value of k that minimizes the probability of error.

(b) What is the gain for the case $\sigma_1^2=1$ and $\sigma_2^2 = 2$, when either $k = 1$ or k is the optimum value?

3. Consider the basic time-series prediction problem where the task is to predict $x(n+p)$ using two "previous" samples $x(n)$ and $x(n-1)$ (a two-tap, p-step predictor). For simplicity, the sequence $x(n)$ is assumed to be wide-sense stationary (WSS) and we use a simple linear predictor of the form

$$\hat{x}(n+p) = w_0 x(n) + w_1 x(n-1) = \mathbf{w}^T \mathbf{x}(n)$$

where $\mathbf{w} = [w_0 \;\; w_1]^T$ and $\mathbf{x}(n) = [x(n) \;\; x(n-1)]^T$. Determine the minimum mean-squared error (MSE) solution $\mathbf{w}_{\mathrm{MSE}}$ for the predictor coefficients. If $x(n)$ is a white sequence, what kind of a predictor you get? Explain. Based on this, what can you say about the predictability of (*i*) white noise and (*ii*) white Gaussian noise, in general ?

4. Explain the main ideas of the BCJR algorithm (i.e., optimum, trellis-based SISO MAP detection/decoding). Which optimality criterion is this algorithm using? What are the similarities and differences with the Viterbi algorithm? How are the branch and path metrics calculated, and how is the trellis search carried out?

5. Discuss the different approaches in multi-antenna communications. Which different kind of gains can be achieved through the use of multiple antennas? Can the different gains be achieved simultaneously? Explain also the significance of antenna spacing and nature of propagation environment in the choice of multi-antenna approach.