TLT-5906 Advanced Course in Digital Transmission
Digitaalisen siirtotekniikan jatkokurssi

Exam 30.1. 2006

You can answer in English, Finnish, or Swedish.

1. Consider a communication system using the following three signals:

$$s_1(t) = \begin{cases} 1 & \text{for } 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

$$s_2(t) = -s_3(t) = \begin{cases} 1 & \text{for } 0 \le t \le T/2 \\ -1 & \text{for } T/2 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

a) What is the dimensionality of the signal space?

b) Find an appropriate basis for the signal space. (*Hint:* You don't need to use the Gram-Schmidt procedure.)

c) Draw the signal constellation for these signals

d) Derive and sketch the optimal decision regions

e) In case of an AWGN channel with noise power spectral density $N_0/2$, which of the three messages is more vulnerable to errors and why? In other words, which of $P(\text{error} \mid s_i \text{ transmitted})$, i=1, 2, 3, is largest?

2. Multipath intensity profile of a channel is

$$p(\tau) = \begin{cases} \frac{1}{a} e^{-\tau/a}, & \text{when } \tau \ge 0 \\ 0, & \text{otherwise} \end{cases}$$

where $a > 0$.

Determine the spaced-frequency correlation function, $\phi(\Delta f)$, in this case. One alternative way to define the coherence bandwidth in this kind of channel, is to require that $\phi(\Delta f)$ satisfies the following relation

$$|\phi(\Delta f)| > \frac{1}{2}$$

within the coherence bandwidth. Determine the coherence bandwidth defined this way when $a = 1\,\mu\text{s}$.

3. Describe briefly the basic signal estimation / optimum filtering problem. Describe also the basic idea of a causal finite-memory (FIR) Wiener filter. What is the fundamental statistical information that the Wiener filters utilize? Starting from the fixed Wiener filter, what steps/approximations are in general needed to end up in the famous least-mean square (LMS) algorithm?