Examination September 2002

1. (4 points) State the problem of optimal filter design for the backward predictor (model, data available, criterion to be minimized).

2. (8 points)

   (a) Consider the "backward" predictor

   $$\hat{u}(n-2) = au(n)$$

   Compute the optimal value of the parameter a and the variance of the optimum prediction error as functions of autocorrelation values of the process $u(n)$.

   (b) Consider the same problem as in (a), but with the predictor:

   $$\hat{u}(n-1) = au(n)$$

   (c) compare the predictors at (a) and (b). Which of them has the smallest variance of prediction error?

3. (5 points) Describe the effect of filtering by a first order filter the noisy gradient used in LMS. Describe the resulting algorithm in terms of updating the quantity $\Delta W(n)$, the increment in parameters at time n.

4. (7 points) Consider the time varying cost function

   $$J(n) = e(n)^2 + \alpha w(n)^2$$

   where $w(n)$ is the parameter of a FIR(1) filter, $e(n)$ is the estimation error

   $$e(n) = d(n) - w(n)u(n)$$

   $d(n)$ is the desired response, $u(n)$ is the input, and α is a constant. Show that the time update for the parameter vector $w(n)$ is defined by

   $$w(n+1) = (1-\mu\alpha)w(n) + \mu u(n)e(n)$$

   What is the role of the constant α (comment the cases of very large α and very small α).

5. (6 points) Consider a sigmoidal perceptron. Write its model, the training equations and the diagram showing the flow of computations.