800 5530 Adaptive Signal Processing
Examination May 2004

1. (4 points) (a) State the recursive least squares estimation problem (model, data available, criterion to be minimized). Explain the role of the forgetting factor. What is the "memory" of the algorithm if the forgetting factor has the exponential form, $\beta(n,i) = \lambda^{n-i}$?

   (3 points) (b) Find the least squares estimate of w_0 in the very simple model $y(t) = w_0$ when the desired data $d(t)$ is given for $t = 1, \ldots, N$ (therefore the input is assumed $u(t) = 1$ for $t = 1, \ldots, N$). What is the significance of the estimate?

   (3 points) (c) Find the recursive least squares solution $w_0(N)$ for the model at (b). Find also the exponential weighted solution ($\beta(n,i) = \lambda^{n-i}$).

2. (3 points) Consider the predictor of order 1

$$\hat{u}(n) = au(n-2)$$

   Compute the optimal value of the parameter a, as a function of autocorrelation values of the process $u(n)$.

3. (3 points) Describe the effect of filtering by a first order filter the noisy gradient used in LMS. Describe the resulting algorithm in terms of updating the quantity $\Delta W(n)$, the increment in parameters at time n.

4. - (4 points) The normalized LMS (NLMS) algorithm can be derived as a constrained algorithm for fulfilling the equality $\underline{w}(n)^T \underline{u}(n) = d(n)$. What is the constraint and describe the transformation of the constrained problem into an unconstrained one (the full derivation of NLMS is not needed here). Write the normalized LMS algorithm for the FIR filter with two paramteres, w_0 and w_1.
   - (2 points) How the algorithm will evolve if the input is $u(0) = 0, u(1) = 0, u(2) = 1, u(3) = 1, u(4) = u(5) = u(6) = \ldots = 0$ and the desired input is $d(0) = 0, d(1) = 0, d(2) = 0, d(3) = 1, d(4) = d(5) = d(6) = \ldots = 0$ (consider different situations for the initial weights).

5. (4 points) Consider a sigmoidal perceptron. Write its model, the training equations and the diagram showing the flow of computations.

6. (4 points) Application description: Draw the structure of an adaptive echo canceller. Discuss the significance of each signal. Which of the two algorithms, LMS or RLS are better suited for this application?