8001302 Signal Compression
Examination on April 2004

1. a) (2 points) How a binary tree can be encoded? (this is needed e.g. for sending to the decoder the information about which prefix code was used for coding) Use an example of tree with $n = 7$ leaves.

   b) (2 points) Use your answer at point a) to give an estimate of the number of binary trees with n leaves.

2. (3 points) Which is the length of the Shanon's code for a symbol j having probability p_j? Specify the Shanon code for the three symbol alphabet, having probabilities $p_1 = 0.51, p_2 = 0.25, p_3 = 0.24$ ($\log_2 0.51 = -0.9714 : \log_2 0.25 = -2; \log_2 0.24 = -2.0589$).

3. (4 points) For a given independent binary source, having Bernoulli distribution with $P(0) = 0.4, P(1) = 0.6$, design a Tunstall tree with 6 leaves. What is the rate of the code?

4. (4 points) Decode the string 1110110010000, knowing it is the Elias codeword for an integer.

   Hint: Decoding is done by inverting the encoding process. The encoding (construction of Elias codewords) is as follows: Let the binary string $B(j)$ consist of the binary expansion of j with the leftmost entry removed (the leftmost entry is always a 1 and is superfluous). Denote $1b_1b_2 \ldots b_{k-1}b_k$ the expansion in binary of the length $|B(j)|$. (attention, not of $B(j)$). Then the Elias codeword of j is

$$E(j) = \begin{cases} 1b_1b_1b_2b_2 \ldots b_{k-1}b_{k-1}01B(j) & if \quad b_k = 0 \\ 1b_1b_1b_2b_2 \ldots b_{k-1}b_{k-1}10B(j) & if \quad b_k = 1 \end{cases}$$

5. (5 points) a) Find the binary Huffman code for the source with symbols $\{a, b, c, d, e\}$ and probabilities $\left(\frac{7}{15}, \frac{1}{5}, \frac{1}{15}, \frac{2}{15}, \frac{2}{15}\right)$. What is the average length of the code?

   b) Draw the canonical Huffman code for the source. Write the code to be transmitted for the string *aabed* using the designed canonical Huffman code (we suppose the decoder has the code tree already available) .

6. (5 points) The source alphabet is known to have 7 symbols, with the probabilities specified in the left table below. What will be the encoder's output for transmitting "EXAM ", such that the original string can be decoded. The encoder started to compute the interval, as shown in the table below.

Character	Probability	Range
SPACE	1/10	0.00 - 0.10
A	3/10	0.10 - 0.40
B	1/10	0.40 - 0.50
E	2/10	0.50 - 0.70
N	1/10	0.70 - 0.80
M	1/10	0.80 - 0.90
X	1/10	0.90 - 1.00

New Character	Low value	High Value
	0.0	1.0
E	0.5	0.7
X		
A		
M		
SPACE		

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