80509 DIGITAL LINEAR FILTERING I

Final Examination 4.3.2002

NO literature in the examination, short, compact, and pithy answers are preferred.

1. Explain shortly (using formulas and/or words) the meanings of the following terms: 1) Difference equation, 2) Direct-form structure, 3) Cascade-form structure, 4) Transposed form structure, 5) Minimax error, 6) Filter stability, 7) Amplitude response, 8) Phase delay and group delay responses, 9) Zero-phase frequency response, 10) Classical recursive digital filters, 11) Rounding and truncation, 12) Filter scaling.

2. (a) Under certain conditions an FIR filter is defined to have a linear phase (four cases). What are these conditions in terms of the impulse response of the filter?
   (b) Consider an FIR filter of order 4. The impulse-response coefficients of this filter are given by $h(0) = h(4) = 1/9$, $h(1) = h(3) = 2/9$, $h(2) = 3/9$. Express the frequency response of this filter in the simplest possible form. What are the phase and group delay responses of this filter? What is the value of the amplitude response at the zero frequency?

3. The traditional techniques for designing linear-phase FIR filters are based on the use of windowing and the Remez multiple exchange algorithm. What are the basic ideas behind these techniques? Compare these techniques with each other.

4. (a) How to use the bilinear transformation for designing recursive digital filters with the aid of analog filters? Why is the bilinear transformation a good technique for generating recursive digital filters?
   (b) Design a first-order Butterworth digital filter with the aid of the bilinear transformation in such a way that the resulting magnitude squared function achieves the value of unity at $\omega = 0$ and the value of $1/2$ ($A_p = 3.0103$ dB) at the passband edge $\omega_p = \pi/2$.

5. It is desired to implement the transfer function

$$H(z) \quad \frac{K}{1+0.8z^{-1}}$$

using two's complement arithmetic. The data wordlength is 1+8 bits. Determine the largest value for K (implemented before the feedback loop) for which there are no overflows (the worst-case scaling). What is the variance of the output noise due to the multiplication roundoff errors?