SGN-2206 Adaptive Signal Processing
Examination December 2005

1. (4 points) State the problem of optimal filter design for the backward predictor (model, data available, criterion to be minimized).

2. (4 points) Consider a sigmoidal perceptron. Write its model, the training equations and the diagram showing the flow of computations.

3. (4 points)
   - Write the normalized LMS algorithm for a FIR filter with two weights.
   - Use an example to show explicitly the computations required for the first two iterations of the algorithm.

4. (3 points) Consider the predictor

   $$\hat{u}(n) = au(n-2)$$

   Compute the optimal value of the parameter a, as a function of autocorrelation values of the process $u(n)$.

5. (5 points) Consider a FIR filter $y(n) = \underline{w}^T \underline{u}(n)$, and denote the noisy gradient estimate used in LMS as $\underline{g}(n) = \hat{\nabla}_{\underline{w}} J$.

   Describe the effect of filtering $\underline{g}(n)$ by a first order IIR filter $H(z) = \frac{1-\gamma}{1-\gamma z^{-1}}$.

   Describe the resulting algorithm in terms of updating the quantity $\Delta \underline{w}(n) = \underline{w}(n) - \underline{w}(n-1)$, the increment in parameters at time n and explain the name momentum LMS given to the resulting algorithm.

6. (6 points) Consider a FIR(1) filter $y(n) = w(n)u(n)$ where all quantities are scalars. We intend to minimize the time varying cost function

   $$J(n) = e(n)^2 + \alpha w(n)^2$$

   where $e(n)$ is the estimation error

   $$e(n) = d(n) - w(n)u(n)$$

   $d(n)$ is the desired response, $u(n)$ is the input, and α is a constant. Show that the time update for the parameter vector $w(n)$ is defined by

   $$w(n+1) = (1 - \mu\alpha)w(n) + \mu u(n)e(n)$$

   What is the role of the constant α (comment the cases of very large α and very small α).

7. (4 points) Application description: Draw the structure of an adaptive echo canceller. Discuss the significance of each signal.