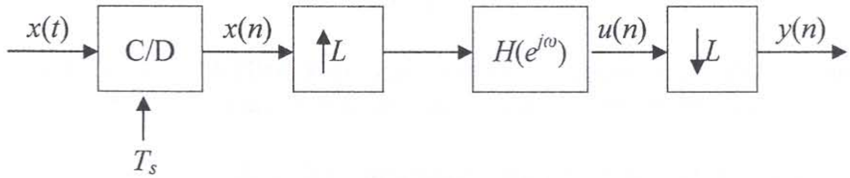
3. (6 points) Consider the following system:



Assume that $X(j\Omega) = 0$ for $|\Omega| > \pi/T_s$ and that

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega} & |\omega| \le \dfrac{\pi}{L} \\ 0 & \dfrac{\pi}{L} < |\omega| \le \pi \end{cases}$$

How is the output of the discrete-time system, $y(n)$, related to the input signal $x(t)$?

4. (3 points) Find the region of convergence of the z-transform of each of the following sequences:

(a) (1 point) $x(n) = \left[\left(\dfrac{1}{2}\right)^n + \left(\dfrac{3}{4}\right)^n\right] u(n-10)$

(b) (1 point) $x(n) = \begin{cases} 1 & -10 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$

(c) (1 point) $x(n) = 2^n u(-n)$

5. (6 points) The deterministic autocorrelation sequence $r(n)$ of a sequence $x(n)$ is defined as

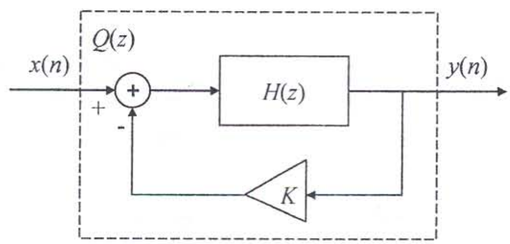
$$r(n) = \sum_{k=-\infty}^{\infty} x(k)x(n+k)$$

(a) (2 points) Express $r(n)$ as the convolution of two sequences and find the z-transform of $r(n)$ in terms of the z-transform of $x(n)$.
(b) (4 points) If $x(n) = a^n u(n)$ where $|a| < 1$, find the autocorrelation sequence $r(n)$ through its z-transform.

6. (5 points) Suppose that we have an unstable second-order system

$$H(z) = \frac{1}{1 - 2z^{-1}}$$

that we would like to stabilize with a feedback system, as shown below:



Find the system function of the closed-loop system, $Q(z)$, and determine the values for the constant feedback gain K that result in a stable system. Assume that K is a real number.