MAT-04006 Engineering Mathematics 123 / Hirvonen

Final Exam 20.12.2018

No calculators, books or lecture notes. A sheet of formulas is printed on a separate paper.

Questions 1-3 are from the first period part of the course, questions 4-6 from the second. Explain your reasoning in the solutions.

1. (a) Find the real and imaginary parts of

$$\frac{1}{2} - \frac{5-8i}{3-4i}.$$

   (b) Solve the equation

$$\sin(2x) = \cos x.$$

2. Consider the vector equation $\mathbf{r} = (13, 8, 1) + t(2, 2, -1)$, where $t \in \mathbb{R}$ and $\mathbf{r} = (x, y, z)$ is any point on the line l. Let $A = (3, -2, 6)$ and $P = (-p, 0, 2p)$, where p is a constant.

   (a) Show that the point A is on the line l.

   (b) Find all values p such that the line AP is perpendicular to the line l.

   (c) Let $p = 1$. Find a unit vector $\hat{\mathbf{n}}$ that is perpendicular to both l and AP.

3. Find the eigenvalues and (all) eigenvectors of

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}.$$

4. (a) Is $(p \vee q) \wedge \widetilde{(\tilde{p} \wedge q)} \leftrightarrow p$ a tautology? Prove your answer.

   (b) Find the solution for

$$\frac{dx}{dt} - 2t(2x - 1) = 0, \quad x(0) = 0.$$

5. Find the interval of convergence for

$$\sum_{k=5}^{\infty} \frac{(-3x)^{k+1}}{2k-1}.$$

6. Use the partial fractions method of integration to calculate

$$\int_{-1}^{0} \frac{x^2 - 9x + 14}{(x+2)(x-1)^2} dx.$$