SGN-11006, Basic Course in Signal Processing

Responsible teachers: Prof. Atanas Gotchev

Exam - 12.12.2016

Use of calculators: **The use of a personal programming calculator is allowed**

You can take the questions with you.

Instructions: Don't forget to write your name on **every** sheet in CAPITAL LETTERS and your student number as well. The total amount of points is 100.

1. (16 points) An L-th order moving average filter is a system that, for an input $x[n]$ produces the output:

$$y[n] = \frac{1}{1+L}\sum_{k=0}^{L} x[n-k]$$

(a) Is the system linear? (b) Is it time invariant? (c) Is it stable? (d) Is it causal? (e) Can the system be represented by its impulse response? **Justify your answers.**
Find the system's (f) impulse response $h[n]$ and (g) frequency response $H(e^{j\omega})$.

2. (16 points) Consider the interconnection of LTI systems show in Figure 1.

(a) Express the frequency response of the overall system $H(e^{j\omega})$ in terms of the frequency responses of the subsystems $H_1(e^{j\omega})$, $H_2(e^{j\omega})$, $H_3(e^{j\omega})$, $H_4(e^{j\omega})$.

(b) Determine the frequency response of the overall system if:

$$h_1[n] = \delta[n]+\delta[n-1]+\delta[n+1]$$
$$h_2[n] = (0.2)^n\mu[n]$$
$$h_3[n] = 2^n\mu[-n]$$
$$h_4[n] = -\delta[n+1]$$

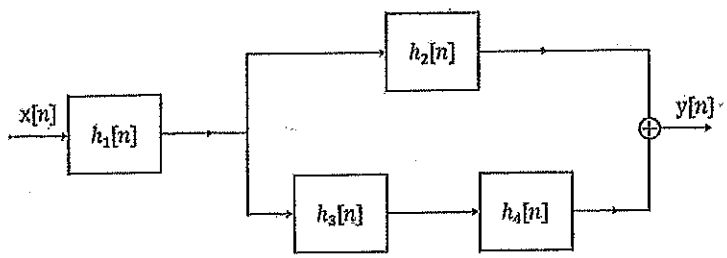

Figure 1: Interconnection of LTI systems.

3. (14 points) A continuous-time signal $x_a(t)$ is to be filtered to remove frequency components in the range 7.5kHz $\leq f \leq$ 15kHz. The maximum frequency component present in the signal is 30kHz. If the signal is sampled at the Nyquist rate, what range of digital frequencies should be rejected by the filter? Sketch the magnitude characteristics of the ideal filter required for this purpose.

4. (20 points) The even samples of the 11-point DFT of a length 11 real sequence are given by $X[0]=-12.6$, $X[2]=2.5-j19.2$, $X[4]=-12.4+j12.7$, $X[6]=-7.5+j13.7$, $X[8]=-3.3+j3.7$, $X[10]=1.5-j5.3$. Find the following:

(a) Determine the remaining five samples

(b) $x[0]$

(c) $\sum_{n=0}^{10} x[n]$

(d) $\sum_{n=0}^{10} |x[n]|^2$

(e) $\sum_{n=0}^{10} e^{j(8\pi n/11)} x[n]$

5. (16 points) Use the Z-transform to perform the convolution of the following two sequences:

$$\begin{aligned} h[n] &= \left(\tfrac{1}{2}\right)^n (\mu[n] - \mu[n-3]) \\ x[n] &= \delta[n+1] + \delta[n-1] \end{aligned}$$

6. (18 points) If the input to a LTI discrete-time system $x[n]$ is

$$x[n] = -(0.1)^n \mu[-n-1]$$

the output is

$$y[n] = \left(\left(-\frac{1}{3}\right)^n + 2\left(\frac{1}{4}\right)\right)^n \mu[n-2]$$

(a) Find the transfer function, $H(z)$, of the system. Plot the poles and zeros of $H(z)$ and indicate the region of convergence.

(b) Is the system stable? Is it causal? Is it minimum phase? **Justify your answer.**