

General instructions

The use of books, notes, calculators, electronic document readers and Turing machines is allowed.

Provide concise answers with short and precise justification.

Write your family name on every page you hand in,

number consecutively all pages that you hand in,

write on the first page the number of pages that you hand in.

No page should contain answers or fragments of answers to more than one question.

Please use pen or dark pencil to write your answers.

Questions:

1. Let 0 denote the empty set. Is $\{\{0\},\{\{0\}\}\}$ an ordinal?
2. Is it true that if a set S has an injective but not surjective map to itself, then S also has a surjective but not injective map to itself?
3. Is there an infinite ordinal whose successor is a cardinal?
4. List explicitly all well formed formulas A of predicate logic with a single binary relational symbol $\$$ which have length at most 7, no propositional symbol occurrence, and a single variable x (occurring or not occurring or occurring any number of times).
5. Let p be a propositional symbol. Consider those formulas of predicate logic of length 8, in which p has a single occurrence and in which no predicate symbol of arity 1 or more has any occurrence. Are there more than 1000 such formulas?

JOKER (optional, replaces any of the above questions):

Design a Turing machine that decides if a number - given in primitive Roman notation, i.e. 7 is given as VIIII – is odd or even.