

No books, no calculators. Please answer **four** of the following numbered questions, on a separate paper. Please use a maximum of one page per answer.

Suomeksi vastaaminen on sallittua: Vastaa neljään seuraavista kysymyksistä; käytä yhteen tehtävään vastaamiseen korkeintaan yksi sivu. Ei kirjoja tai laskimia.

1. Let  $\Sigma = \{a, b, c\}$ . For the following languages over  $\Sigma$ , draw a DFA or, if not possible, a PDA, and if this is not possible either, then a Turing machine.

- (a)  $\{a^n b^m c^n \mid n \geq 0, m \geq 0\}$
- (b)  $\{a^n b^m \mid n + m \equiv 0 \pmod{3}\}$
- (c)  $\{a^n b^{n+m} c^m \mid n \geq 0 \wedge m \geq 0\}$

(A correct automaton is worth one point, and one point is awarded for getting the correct type of automaton. Half points are possible.)

2. Pick one of the following and explain what it means. Use formal details and examples.

- (a) A Turing machine that can print out its own description
- (b) Church-Turing Thesis
- (c) Savitch theorem

3. Pick one of the following and explain. Use details and examples.

- (a) Four *complete* complexity classes with at least four examples from different classes.
- (b) Four different kinds of automata and three kinds of languages they can decide.

4. A linear bounded automaton (LBA) is a specific kind of Turing machine. The decision problem "Does the LBA  $M$  accept the word  $\alpha$ ?" is decidable, but the problem "Is the language accepted by the LBA empty?" is not. How is this possible? Explain.

5. For each of the following, indicate whether the language is in **P**, or if it is **NP**-complete. If it is the latter, then prove by reduction from *SAT* or *3SAT*.

- (a) *HP* is the language of graphs ( $G$ ) where  $G$  is a graph that has a path that visits every node exactly once.
- (b) *2SAT* is the language of satisfiable propositional formulas that are of the form  $(x_1 \vee y_1) \wedge \dots \wedge (x_n \vee y_n)$ , where each  $x_i$  is a literal (i.e., a propositional symbol or a negation of one).

6. Prove that the following problems are undecidable. For instance, use reduction from  $A_{TM}$

- (a)  $E_{TM} = \{(M) \mid \text{the language accepted by } M \text{ is empty}\}$
- (b)  $SS_{TM} = \{(M, M') \mid M \text{ accepts every string that } M' \text{ accepts}\}$  (I.e., the language of  $M$  is a subset of the language of  $M'$ )