

No calculator, no extra material is allowed!
 Return the distributed material clean!

In case you are take an examination of

- the data mining part, solve problems 1 - 4,
- the many-valued similarity part, solve problems 5 - 8,
- the whole course, solve even problems 2, 4, 6 and 8.

Problem 1.

Let M and N be two models that generates the following two four-fold tables.

M	ψ	$\neg\psi$	N	ψ	$\neg\psi$
ϕ	a_1	b_1	ϕ	a_2	b_2
$\neg\phi$	c_1	d_1	$\neg\phi$	c_2	d_2

Under which conditions N is (a) associational, (b) implicationally better than M ? (c) Define the truth condition of Σ -double implication quantifiers.

Problem 2.

Is $\neg\phi$ a logical consequences of a set $\{\psi \vee \neg\phi, \psi \wedge \phi\}$?

Problem 3.

Prove that founded equivalence quantifiers are associational.

Problem 4.

(a) Why are rules of inference useful in GUHA-logic framework? (b) Let $\phi(x)$, $\psi(x)$, $\chi(x)$ be formulae, and let \approx be an implicational quantifier. Prove that

$$\frac{[\phi \wedge \neg\chi] \approx \psi}{\phi \approx [\chi \vee \psi]}$$

is a sound rule of inference. Does this claim hold is \approx is not implicational?

Problem 5.

Let L be an MV-algebra. Prove that a Galois connection, also called *residuation*

$$a \odot b \leq c \text{ if, and only if } a \leq b \rightarrow c$$

holds for all $a, b, c \in L$.

Problem 6.

Let L be the Lukasiewicz structure. (a) How are the operations $\rightarrow, *, \wedge, \vee, \odot, \oplus, \leftrightarrow$ defined on L ? (b) What is their logic interpretation?

Problem 7.

J. S. Mill defined in 1843: if two objects A and B agree on k attributes and disagree on m attributes, then the number

$$\text{sim}(A, B) = \frac{k}{k+m}$$

can be taken to measure the degree of *partial identity* between A and B . Prove that $\text{sim}(\neg, \neg)$ is a Lukasiewicz many-valued similarity.

Problem 8.

Consider a fuzzy rule base system presented apart. Assume pedestrian waiting time is 15 sec, there are 2 approaching vehicles and the shortest gap between them is 1 sec. The corresponding weights are 1, 2 and 3, respectively. Moreover, in 50-50 situations pedestrians are winners. Calculate the corresponding output by total fuzzy similarity method. - It is enough to estimate the membership degrees from the picture; you may do drawings and sketches on that sheet of paper. Enclose the sheet with your answers.