APPENDIX A, Equations

1. WGS 84 fundamental parameters

Ellipsoid (earth model)

a = 6378137 m, semi-major axis

$1/f$ = 298.257223563, reciprocal flattening

Earth's angular velocity, $\omega_E = 7292115.0 \times 10^{-11}$ rad/s

Earth's gravitational constant, $GM = 3986004.418 \times 10^8$ m^3/s^2

Speed of light in a vacuum, $c = 2.99782458 \times 10^8$ m/s

2. Equations

$$D = f_R - f_T = -\frac{f_T}{v_s}\dot{r} = -\frac{\dot{r}}{\lambda}$$

v_s = speed of propagation of the waves, λ = wave length of the signal, $\dot{r}$ = range rate

$$x_m = x + b$$
$$x_m = (1+a)x$$

x_m = measured value, x = true value, b = bias error, a = scale factor error

$$e^2 = (a^2 - b^2)/a^2$$
$$f = (a-b)/a$$
$$e^2 = 2f - f^2$$

a = length of semi-major axis, b = length of semi-minor axis, e = eccentricity, f = flattening

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$
$$x = (N+h)\cos\phi\cos\lambda$$
$$y = (N+h)\cos\phi\sin\lambda$$
$$z = \left(N(1-e^2)+h\right)\sin\phi$$

Coordinate conversion from geodetic (ellipsoidal) to Cartesian, where latitude $=\phi$, longitude $=\lambda$, height $=h$

$$\mathbf{R}_L = \begin{bmatrix} -\sin\lambda_0 & \cos\lambda_0 & 0 \\ -\sin\phi_0\cos\lambda_0 & -\sin\phi_0\sin\lambda_0 & \cos\phi_0 \\ \cos\phi_0\cos\lambda_0 & \cos\phi_0\sin\lambda_0 & \sin\phi_0 \end{bmatrix}$$

ECEF to ENU, rotation matrix $\mathbf{R}_L$ the origin of the ENU frame: latitude ϕ_0 and longitude λ_0

$$\tan(az) = \frac{x_E}{x_N}$$
$$\sin(el) = \cos(ze) = \frac{x_U}{\sqrt{x_E^2 + x_N^2 + x_U^2}}$$

Azimuth, zenith and elevation angles, defined in ENU frame

$$F = \frac{f - f_0}{f_0} = \frac{\Delta f}{f_0}$$
$$\Delta f = f - f_0$$

F = relative frequency error = normalized frequency error
Δf = frequency deviation

$$r^{(k)} = \sqrt{\left(x^{(k)} - x\right)^2 + \left(y^{(k)} - y\right)^2 + \left(z^{(k)} - z\right)^2} = \left\|\mathbf{x}^{(k)} - \mathbf{x}\right\|$$ User-to-satellite geometric range

$$\text{GDOP} = \sqrt{\text{H}_{11} + \text{H}_{22} + \text{H}_{33} + \text{H}_{44}}$$
$$\mathbf{H} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{G}\right)^{-1}$$
$\mathbf{G}$ = geometry matrix

$$\mathbf{G} = \begin{bmatrix} \left(-\mathbf{1}^{(1)}\right) & 1 \\ \vdots & \vdots \\ \left(-\mathbf{1}^{(K)}\right) & 1 \end{bmatrix}$$ Geometry matrix

$$\left(-\mathbf{1}^{(k)}\right) = \left[-\frac{x^{(k)} - x_0}{r_0^{(k)}}, \quad -\frac{y^{(k)} - y_0}{r_0^{(k)}}, \quad -\frac{z^{(k)} - z_0}{r_0^{(k)}}\right]$$

$$\mathbf{x} = \left(\mathbf{B}^{\mathrm{T}}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{y}$$ Least squares solution of $\mathbf{y} = \mathbf{B}\mathbf{x}$, where $\mathbf{B}^{\mathrm{T}}\mathbf{B}$ is invertible

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$ Inverse of 2x2 matrix $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

APPENDIX B, code generation

Table 3-I. Code Phase Assignments (sheet 1 of 2)

SV ID No.	GPS PRN Signal No.	Code Phase Selection		Code Delay Chips		First 10 Chips Octal* C/A	First 12 Chips Octal P
		C/A($G2_i$)****	($X2_i$)	C/A	P		
1	1	2 ⊕ 6	1	5	1	1440	4444
2	2	3 ⊕ 7	2	6	2	1620	4000
3	3	4 ⊕ 8	3	7	3	1710	4222
4	4	5 ⊕ 9	4	8	4	1744	4333
5	5	1 ⊕ 9	5	17	5	1133	4377
6	6	2 ⊕ 10	6	18	6	1455	4355
7	7	1 ⊕ 8	7	139	7	1131	4344
8	8	2 ⊕ 9	8	140	8	1454	4340
9	9	3 ⊕ 10	9	141	9	1626	4342
10	10	2 ⊕ 3	10	251	10	1504	4343
11	11	3 ⊕ 4	11	252	11	1642	|
12	12	5 ⊕ 6	12	254	12	1750	|
13	13	6 ⊕ 7	13	255	13	1764	|
14	14	7 ⊕ 8	14	256	14	1772	|
15	15	8 ⊕ 9	15	257	15	1775	|
16	16	9 ⊕ 10	16	258	16	1776	|
17	17	1 ⊕ 4	17	469	17	1156	|
18	18	2 ⊕ 5	18	470	18	1467	|
19	19	3 ⊕ 6	19	471	19	1633	4343

* In the octal notation for the first 10 chips of the C/A code as shown in this column, the first digit (1) represents a "1" for the first chip and the last three digits are the conventional octal representation of the remaining 9 chips. (For example, the first 10 chips of the C/A code for PRN Signal Assembly No. 1 are: 1100100000).

** C/A codes 34 and 37 are common.

*** PRN sequences 33 through 37 are reserved for other uses (e.g. ground transmitters).

**** The two-tap coder utilized here is only an example implementation that generates a limited set of valid C/A codes.

⊕ = "exclusive or"

NOTE: The code phase assignments constitute inseparable pairs, each consisting of a specific C/A and a specific P code phase, as shown above.