MAT-41196 Graph Theory Examination 13.12.2012

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NB This is a closed-book exam, no material is allowed. Nonprogrammable calculators are allowed.

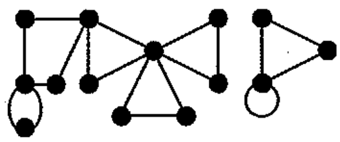
1. The *degree* of an edge e in a graph is the number of edges adjacent to e. A graph is said to be *d-line-regular* if it is simple and each of its edges has degree d.

a) The complete graph K_n is d_n-line-regular for some degree d_n. What is this d_n? (Here we naturally assume that $n \geq 2$.)

b) Find all 3-line-regular graphs with the smallest possible number of vertices. Explain your reasoning!

(As is easily seen, complete graphs give the single line-regular graphs with the fewest vertices when the degree is even. The case of odd degrees is then different. There are many other names for line-regularity in the litterature!)

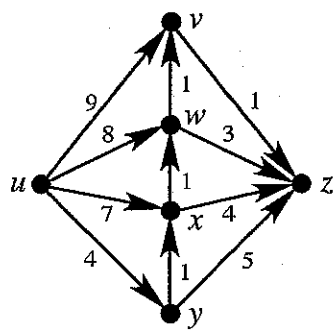
2. In graph G every edge is in exactly one circuit. We identify here a circuit with the subgraph induced by its edges. An example of such a graph is on the right.



For graphs of this kind, the number of circuits depends only on the numbers of vertices, edges and components. If G has n vertices, m edges, and k components, then how many circuits does it have? Explain your reasoning! (*Hint:* Consider first the case of a connected G.)

3. **a)** Dijkstra's Algorithm, how it works and what is its function.

b) Apply Dijkstra's Algorithm to the digraph on the right: starting vertex u, goal vertex z, arc weights are given in the figure. Explain carefully each step you take! (The solution is of course immediate, this is just to see you know how to use the algorithm.)



4. Transport networks, maximum flows, and the Ford–Fulkerson Algorithm.

5. Explain what is **a)** planarity of a graph, **b)** planar embedding of a graph, **c)** coloring of a graph, and **d)** how they are connected.

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