TAMPERE UNIVERSITY OF TECHNOLOGY
Department of Software Systems

OHJ-2306 **Introduction to Theoretical Computer Science**   Examination
prof. Tapio Elomaa   Dec. 10, 2012

Write your name and student number to each separate answer sheet. Neither calculators nor any other extra material is allowed in the exam.

All students are required to answer questions 1 and 2. In addition, you may choose to answer any two questions from among 3–6. The maximum score for questions 1 and 2 is 8 points and for questions 3–6 7 points. In total 30 points.

Give careful and detailed answers to the questions!

ANSWER QUESTIONS 1 AND 2

1. Let *ABBA* be the language

   $\{\, w \in \{a, b\}^* \mid w$ has equal numbers of occurences of substrings ab and $ba \,\}$.

   Give well-grounded constructive answers to the following questions.

   (a) Is *ABBA* a regular language?
   (b) Can language *ABBA* be generated by a right-linear context-free grammar?
   (c) Can language *ABBA* be generated by an unrestricted context-free grammar?

2. Is the following language decidable?

   $$A_{\text{TM}} = \{\, \langle M, w \rangle \mid M \text{ is a Turing machine that accepts } w \,\} .$$

   Use the fact that the "diagonal language" $D = \{\, b \in \{0, 1\}^* \mid b \notin L(M_b) \,\}$ is not Turing-recognizable. Be careful in the details of the proof.

ANSWER TWO OF QUESTIONS 3–6

3. Explain about context-free grammars, ambiguity, derivations, and parse trees.

4. Give a standard Turing machine for recognizing the language

$$\{\, w \in \{0,1\}^* \mid w \text{ contains equally many 0s and 1s} \,\}.$$

5. Define incompressibility of binary strings and the required related concepts. Show that incompressible strings of every length exist.

6. Show that CSAT $\leq_m^p$ 3SAT.