

Only the calculator of the faculty may be used in the exam.  
 You can take the question paper with you.

1. Do nonlinear filters commute, i.e., is the filtering result after filtering the signal first by the filter  $f(\cdot)$  and then the result by the filter  $g(\cdot)$  the same as after filtering the signal first by the filter  $g(\cdot)$  and then the result by the filter  $f(\cdot)$ ? Justify your answer. (6 points)
2. Show that
  - a) the LH+ filter preserves vertical and horizontal lines but not diagonal lines. (2 points)
  - b) the R1LH+ filter preserves diagonal lines but not vertical and horizontal lines. (2 points)
  - c) the 2LH+ filter preserves vertical, horizontal and diagonal lines. (2 points)
3. Let the input sample be  $\mathbf{x} = (1, 2, 9, 8, 4, 11)$  and the rank selection rule is given by

$$S(R(X_1), R(X_2), R(X_3)) = \begin{cases} 3, & \text{if } (R(X_1), R(X_2), R(X_3)) = (1, 2, 3), \\ 3, & \text{if } (R(X_1), R(X_2), R(X_3)) = (1, 3, 2), \\ 1, & \text{if } (R(X_1), R(X_2), R(X_3)) = (2, 1, 3), \\ 3, & \text{if } (R(X_1), R(X_2), R(X_3)) = (2, 3, 1), \\ 2, & \text{if } (R(X_1), R(X_2), R(X_3)) = (3, 1, 2), \\ 1, & \text{if } (R(X_1), R(X_2), R(X_3)) = (3, 2, 1), \end{cases}$$

Filter the signal by permutation filter defined by the rank selection rule  $S$ . No appending is required. (6 points)

4. Let  $\mathbf{x}$  be the  $3 \times 3$  input

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix},$$

where each  $x_i \in \{0, 1\}$  and  $x_5$  is the center sample. Consider the (binary) stack filter:

$$f(\mathbf{x}) = x_4 x_5 x_6 + (x_4 + x_5 + x_6) \cdot m^{3 \times 3}(\mathbf{x}),$$

where  $m^{3 \times 3}(\mathbf{x})$  is a Boolean function representing the  $3 \times 3$  median. Show that this filter preserves horizontal white lines (1's), horizontal black lines (0's) and if there is no such line, it simply takes the median of the whole window. (6 points)

5. On the right is an original signal and below are filtered versions of it. Give the corresponding output S1-S6 for each filter a)-f). Length of the filter window is 5 for all the filters.

- a)  $L$ -filter ( $a = (0.5, 0, 0, 0, 0.5)$ ) (1 point)
- b) LUM sharpener ( $t = 2$ ) (1 point)
- c) 2<sup>nd</sup> ranked-order filter (1 point)
- d) 0.2-trimmed mean filter (1 point)
- e)  $K$ -nearest neighbor filter ( $K = 3$ ) (1 point)
- f) Wilcoxon filter (1 point)

