TLT-6306: RF Equipment for Wireless Networks
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Calculators are allowed in this exam.

Exam 15.12.2011
Answer to all questions!
Total maximum points: 30 (+ exercise bonus)

1. Explain shortly:

a) Noise figure **(2 p)**

b) Grating lobes **(2 p)**

c) Circular polarization **(2 p)**

2. a)
Show by calculation (provide also the explanation) what is the power ratio in decibel scale between the first output port and the input port in:

2-way splitter, 3-way splitter, 4-way splitter.

Assume that the splitters are ideal (e.g. no internal losses). **(2 p)**

b) How does the tapper differ from splitter? Consider a 2-way tapper. What is the power ratio in decibel scale between the output port #2 and the input port, if the power ratio between output port #1 and the input port is set to -5 dB. **(3 p)**

c) Draw/sketch an example of a radio system where you could use these kind of components. **(1 p)**

3. Two collinear, z-directed, and short dipoles are placed at a distance of half wavelength as depicted in Figure 1. The amplitude gain of each element is 1 (i.e., 0 dB) and the feeding phase difference between each element is 0°. Formulate the normalized radiation pattern as a function of elevation angle θ produced by such antenna array.

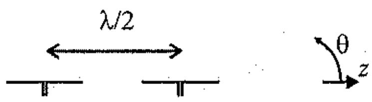

Figure 1

(6 p)

4. a)
Describe and explain three different ways to produce antenna diversity. **(3 p)**

b)
Consider a mobile receiver (operating bandwidth 200 kHz). Calculate, how strong should be the input signal level to the receiver if the signal-to-noise requirement at the output is 9 dB? The receiver noise figure is 7 dB. Assume that the input signal does not experience attenuation when inside the receiver structure. **(3 p)**

5. Explain, how the following parameters affect the radiation properties of linear antenna array

a) Number of array elements (with fixed inter-element spacing) **(2p)**
b) Element spacing **(2p)**
c) Phase difference between the feeding current of different elements **(2p)**

Some (more or less) useful equations:

$$G_a = \frac{p_1}{p_i}, G_b = \frac{p_2}{p_1}, G_c = \frac{p_o}{p_2}$$

$$10\log_{10}\frac{P_1[W]}{1mW}$$

$$\frac{10^{\frac{P[dBm]}{10}}}{1000}$$

$$v_n = \sqrt{\frac{4hfBR}{e^{hf/kT}-1}}$$

$$P_n = \frac{\left(\frac{v_n}{2}\right)^2}{R} = \frac{v_n^2}{4R} = \frac{\left(\sqrt{4kTBR}\right)^2}{4R} = kTB$$

$$\frac{p_o}{p_i} = \frac{p_1}{p_i}\frac{p_2}{p_1}\frac{p_o}{p_2} = G_a G_b G_c$$

$$Y = \frac{P_1}{P_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1$$

$$2f_1 - f_2, \quad 2f_2 - f_1$$

$$F = \frac{CNR_{in}}{CNR_{out}}$$

$$\log_{10}\frac{p_o}{p_i} = \log_{10} G_a + \log_{10} G_b + \log_{10} G_c$$

$$10\log_{10}\frac{p_o}{p_i}$$

$$\lambda = \frac{c}{f}$$

$$\beta = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\varepsilon}$$

$$\operatorname{div}\mathbf{D}(\mathbf{r},t) = \rho(\mathbf{r},t)$$

$$\operatorname{div}\mathbf{B}(\mathbf{r},t) = 0$$

$$A_z = \iiint_{v'} \mu J_z \frac{e^{-j\beta R}}{4\pi R} dv'$$

$$A_z = \int \mu I(z')\frac{e^{-j\beta(r - z'\cos\theta)}}{4\pi r}dz' = \frac{\mu e^{-j\beta r}}{4\pi r}\int I(z')e^{j\beta z'\cos\theta}dz'$$

$$\operatorname{curl}\mathbf{E}(\mathbf{r},t) = -\frac{\partial\mathbf{B}(\mathbf{r},t)}{\partial t}$$

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \ldots + \frac{F_n - 1}{G_1G_2\ldots G_{n-1}}$$

$$R = \frac{2L^2}{\lambda} \qquad R = 0.62\sqrt{L^3/\lambda}$$

$$\operatorname{curl}\mathbf{H}(\mathbf{r},t) = \frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t} + \mathbf{J}(\mathbf{r},t)$$

$$T_{cas} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \ldots + \frac{T_{en}}{G_1G_2\ldots G_{n-1}}$$

0 dBd = 2.15 dBi

$$SSL_{dB} = 20\log_{10}\frac{|F(SSL)|}{|F(\max)|}$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$h = 6.546 \cdot 10^{-34}$ Jsec, Planck's constant
c = 299 792 458 *m/s*, speed of light
$k = 1.38 \cdot 10^{-23}$ J/°K, Boltsmann's constant

$$R_a = R_r + R_l \qquad Z_a = R_a + jX_a \qquad V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$\frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad -20\log_{10}|\Gamma| \text{ dB}$$

$$F(\theta,\phi) = g(\theta,\phi)\cdot f(\theta,\phi)$$

$$D = \frac{U_{\max}}{U_{ave}} \quad \varepsilon_r = \frac{P}{P_{in}} \quad G = \varepsilon_r D$$

$$G(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{in}} \qquad U = \frac{dP}{d\Omega} \qquad P = \frac{1}{2}I^2 R_a$$

$$I(z) = I(0)\sin\left[\beta\left(\frac{L}{2} - |z|\right)\right] \qquad AF = \sum_{n=0}^{N-1} A_n e^{jn\psi} \qquad \psi = \beta d\cos(\theta) + \alpha$$

$$N_{UL\_R} = k\cdot T\cdot B\cdot F_R \cdot G_{T\_UL}$$

$$N_{UL\_BS} = k\cdot T\cdot B\cdot F_{BS}$$

$$N_{UL} = N_{UL\_R} + N_{UL\_BS} = k\cdot T\cdot B\left(F_{BS} + G_{T\_UL}\cdot F_R\right)$$

$$AF = A_0 e^{j(N-1)\psi/2}\frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

$$AF = I_0 e^{-j\xi_0} + I_1 e^{-j\xi_1} + I_2 e^{-j\xi_2} + \cdots I_M e^{-j\xi_M}$$

$$f(\psi) = \frac{\sin(N\psi/2)}{N\sin(\psi/2)} \qquad \mathbf{W} = \frac{1}{L}\mathbf{S}_0$$

$$E(\theta,\phi) = \sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn} e^{j(2\pi/\lambda)(md_x\cos(\theta) + nd_y\sin(\phi)\sin(\theta))}$$

$$C_{SISO} = \log_2\left(1 + \frac{P}{\sigma_n^2}|h(\tau,t)|^2\right) \qquad C_{SIMO} = \log_2\left(1 + \frac{P}{\sigma_n^2}\sum_{i=1}^{N}|h_i(\tau,t)|^2\right)$$

$$C_{MISO} = \log_2\left(1 + \frac{1}{M}\frac{P}{\sigma_n^2}\sum_{j=1}^{M}|h_j(\tau,t)|^2\right) \qquad C_{MIMO} = \sum_{k=1}^{K}\log_2\left(1 + \lambda_k\frac{P_k}{\sigma_n^2}\right)$$

Dolph Chebyshev

Odd N

$$\boxed{E(\theta,\phi) = \sum_{n=0}^{\left(\frac{N-1}{2}\right)} a_n\cos(2nu)}$$

Even N

$$\boxed{E(\theta,\phi) = \sum_{n=0}^{\left(\frac{N-1}{2}\right)} a_n\cos[(2n+1)u]}$$

$$u = \frac{\pi d}{\lambda}\sin(\theta)\sin(\phi)$$

N	Harmonic	$\cos(\mu)$	Equivalent expression
1	$m=0$	$\cos(0)$	1
2	$m=1$	$\cos(u)$	$\cos(u)$
3	$m=2$	$\cos(2u)$	$2\cos^2(u) - 1$
4	$m=3$	$\cos(3u)$	$4\cos^3(u) - 3\cos(u)$
5	$m=4$	$\cos(4u)$	$8\cos^4(u) - 8\cos^2(u) + 1$
6	$m=5$	$\cos(5u)$	$16\cos^5(u) - 20\cos^3(u) + 5\cos(u)$
7	$m=6$	$\cos(6u)$	$32\cos^6(u) - 48\cos^4(u) + 18\cos^2(u) - 1$
8	$m=7$	$\cos(7u)$	$64\cos^7(u) - 112\cos^5(u) + 56\cos^3(u) - 7\cos(u)$

$$\rho_{12} = \mathrm{E}\left[\sum_{i=1}^{n_r} e^{-j\phi_i}\right] = \mathrm{E}\left[\sum_{i=1}^{n_r} e^{-jkd\sin(\theta)}\right] \qquad J_0(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{jx\cos(\alpha)}d\alpha$$

$$\rho_{12} = \int_0^{2\pi} p(\theta)e^{-jkd\sin(\theta)}d\theta \qquad \rho(d) = \frac{1}{2\pi}\int_0^{2\pi} e^{-jkd\sin(\theta)}d\theta = \left|J_0\left(\frac{2\pi d}{\lambda}\right)\right|$$

$$P_\Gamma(\gamma_t) = \Pr[\Gamma \le \gamma_t] \qquad \Gamma = \max\{\Gamma_1, \Gamma_2, \cdots \Gamma_M\}$$

$$= \Pr[\Gamma_1, \Gamma_2, \cdots \Gamma_M \le \gamma_t] = \prod_{i=1}^{M} P_{\Gamma_i}(\gamma_t)$$

$$T_m(z) = \cos\left(m\cos^{-1}(z)\right) \qquad -1 \le z \le 1$$

$$T_m(z) = \cosh\left(m\cosh^{-1}(z)\right) \qquad |z| > 1$$

$$p(\gamma_i) = \frac{1}{\gamma_0}e^{-\gamma_i/\gamma_0}, \quad \gamma_0 \ge 0 \qquad \gamma_0 = 2\sigma^2\frac{E_b}{N_0}$$

$$\log_a x = y \Leftrightarrow a^y = x \qquad \sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$P_\Gamma(\gamma) = \Pr[\Gamma \le \gamma] = \Pr[\max\{\Gamma_i \le \gamma\}]$$

$$= \Pr[\Gamma_1, \Gamma_2, \cdots \Gamma_M \le \gamma] = \left(1 - e^{\gamma_i/\gamma_0}\right)^M \qquad P(\gamma) = 1 - e^{-\gamma/\gamma_0}\sum_{i=1}^{M}\frac{1}{(i-1)!}\left(\frac{\gamma}{\gamma_0}\right)^{i-1}$$

$$\log_a x = \frac{\log_b x}{\log_b a} \qquad \cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$