

1. Explain shortly:

a) Noise figure
(2 p)

b) Grating lobes
(2 p)

c) Circular polarization
(2 p)

2.

a)
Show by calculation (provide also the explanation) what is the power ratio in decibel scale between the first output port and the input port in:

2-way splitter,

3-way splitter,

4-way splitter.

Assume that the splitters are ideal (e.g. no internal losses). (2 p)

b) How does the taper differ from splitter? Consider a 2-way taper. What is the power ratio in decibel scale between the output port #2 and the input port, if the power ratio between output port #1 and the input port is set to -5 dB. (3 p)

c) Draw/sketch an example of a radio system where you could use these kind of components. (1 p)

3.

Two collinear, z-directed, and short dipoles are placed at a distance of half wavelength as depicted in Figure 1. The amplitude gain of each element is 1 (i.e., 0 dB) and the feeding phase difference between each element is 0° . Formulate the normalized radiation pattern as a function of elevation angle θ produced by such antenna array.

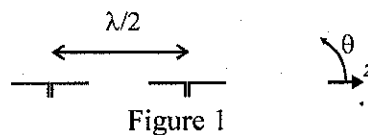


Figure 1

(6 p)

4.

a)
Describe and explain three different ways to produce antenna diversity. (3 p)

b)
Consider a mobile receiver (operating bandwidth 200 kHz). Calculate, how strong should be the input signal level to the receiver if the signal-to-noise requirement at the output is 9 dB? The receiver noise figure is 7 dB. Assume that the input signal does not experience attenuation when inside the receiver structure. (3 p)

5.

Explain, how the following parameters affect the radiation properties of linear antenna array

a) Number of array elements (with fixed inter-element spacing) (2p)

b) Element spacing (2p)

c) Phase difference between the feeding current of different elements (2p)

Some (more or less) useful equations:

$$G_a = \frac{P_1}{P_i}, G_b = \frac{P_2}{P_1}, G_c = \frac{P_o}{P_2}$$

$$10 \log_{10} \frac{P_i [W]}{1mW} \quad \frac{P_i [dBm]}{1000}$$

$$v_n = \sqrt{\frac{4hfBR}{e^{hf/kT} - 1}}$$

$$P_n = \frac{\left(\frac{v_n}{2}\right)^2}{R} = \frac{v_n^2}{4R} = \frac{(\sqrt{4kTBR})^2}{4R} = kTB$$

$$\frac{P_o}{P_i} = \frac{P_1}{P_i} \frac{P_2}{P_1} \frac{P_o}{P_2} = G_a G_b G_c$$

$$Y = \frac{P_1}{P_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1$$

$$2f_1 - f_2, \quad 2f_2 - f_1 \quad F = \frac{CNR_{in}}{CNR_{out}}$$

$$\log_{10} \frac{P_o}{P_i} = \log_{10} G_a + \log_{10} G_b + \log_{10} G_c$$

$$10 \log_{10} \frac{P_o}{P_i}$$

$$\lambda = \frac{c}{f}$$

$$\beta = \frac{2\pi}{\lambda} = \omega \sqrt{\mu\epsilon}$$

$$\text{div } \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t)$$

$$\text{div } \mathbf{B}(\mathbf{r}, t) = 0$$

$$A_z = \iiint_V \mu J_z \frac{e^{-j\beta R}}{4\pi R} dv'$$

$$A_z = \int \mu I(z') \frac{e^{-j\beta(r-z'\cos\theta)}}{4\pi r} dz' = \frac{\mu e^{-j\beta r}}{4\pi r} \int I(z') e^{j\beta z' \cos\theta} dz'$$

$$\text{curl } \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t}$$

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}}$$

$$R = \frac{2L^2}{\lambda}$$

$$R = 0.62 \sqrt{L^3/\lambda}$$

$$\text{curl } \mathbf{H}(\mathbf{r}, t) = \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} + \mathbf{J}(\mathbf{r}, t)$$

$$T_{cas} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \dots + \frac{T_{en}}{G_1 G_2 \dots G_{n-1}}$$

$$0 \text{ dBd} = 2.15 \text{ dBi}$$

$$SSL_{dB} = 20 \log_{10} \frac{|F(SSL)|}{|F(\max)|}$$

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$h = 6.546 \cdot 10^{-34}$ Jsec, Planck's constant

$c = 299\,792\,458$ m/s, speed of light

$k = 1.38 \cdot 10^{-23}$ J/°K, Boltzmann's constant

$$R_a = R_r + R_t$$

$$Z_a = R_a + jX_a$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

$$\frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$-20 \log_{10} |\Gamma| \text{ dB}$$

$$F(\theta, \phi) = g(\theta, \phi) \cdot f(\theta, \phi)$$

$$D = \frac{U_{\max}}{U_{ave}} \quad \epsilon_r = \frac{P}{P_{in}} \quad G = \epsilon_r D$$

$$G(\theta, \phi) = \frac{4\pi U(\theta, \phi)}{P_{in}}$$

$$U = \frac{dP}{d\Omega}$$

$$P = \frac{1}{2} I^2 R_a$$

$$I(z) = I(0) \sin \left[\beta \left(\frac{L}{2} - |z| \right) \right]$$

$$AF = \sum_{n=0}^{N-1} A_n e^{jn\psi}$$

$$\psi = \beta d \cos(\theta) + \alpha$$

$$N_{UL_R} = k \cdot T \cdot B \cdot F_R \cdot G_{T_UL}$$

$$N_{UL_BS} = k \cdot T \cdot B \cdot F_{BS}$$

$$N_{UL} = N_{UL_R} + N_{UL_BS} = k \cdot T \cdot B (F_{BS} + G_{T_UL} \cdot F_R)$$

$$AF = A_o e^{j(N-1)\psi/2} \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

$$AF = I_o e^{-j\epsilon_0} + I_1 e^{-j\epsilon_1} + I_2 e^{-j\epsilon_2} + \dots + I_M e^{-j\epsilon_M}$$

$$f(\psi) = \frac{\sin(N\psi/2)}{N \sin(\psi/2)}$$

$$\mathbf{W} = \frac{1}{L} \mathbf{S}_o$$

$$E(\theta, \phi) = \sum_{m=1}^M \sum_{n=1}^N W_{mn} e^{j(2\pi/\lambda)(md_x \cos(\theta) + nd_y \sin(\theta) \sin(\phi))}$$

$$C_{SISO} = \log_2 \left(1 + \frac{P}{\sigma_n^2} |h(\tau, t)|^2 \right)$$

$$C_{SIMO} = \log_2 \left(1 + \frac{P}{\sigma_n^2} \sum_{i=1}^N |h_i(\tau, t)|^2 \right)$$

$$C_{MISO} = \log_2 \left(1 + \frac{1}{M} \frac{P}{\sigma_n^2} \sum_{j=1}^M |h_j(\tau, t)|^2 \right)$$

$$C_{MIMO} = \sum_{k=1}^K \log_2 \left(1 + \lambda_k \frac{P_k}{\sigma_n^2} \right)$$

Dolph Chebyshev

Odd N

$$E(\theta, \phi) = \sum_{n=0}^{\frac{N-1}{2}} a_n \cos(2nu)$$

Even N

$$E(\theta, \phi) = \sum_{n=0}^{\frac{N-1}{2}} a_n \cos[(2n+1)u]$$

$$u = \frac{\pi d}{\lambda} \sin(\theta) \sin(\phi)$$

$$\rho_{12} = E \left[\sum_{i=1}^n e^{-j\phi_i} \right] = E \left[\sum_{i=1}^n e^{-jkd \sin(\theta)} \right]$$

$$J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{jx \cos(\alpha)} d\alpha$$

$$\rho_{12} = \int_0^{2\pi} p(\theta) e^{-jkd \sin(\theta)} d\theta$$

$$\rho(d) = \frac{1}{2\pi} \int_0^{2\pi} e^{-jkd \sin(\theta)} d\theta = \left| J_0 \left(\frac{2\pi d}{\lambda} \right) \right|$$

$$P_\Gamma(\gamma_i) = \Pr[\Gamma \leq \gamma_i]$$

$$\Gamma = \max\{\Gamma_1, \Gamma_2, \dots, \Gamma_M\}$$

$$= \Pr[\Gamma_1, \Gamma_2, \dots, \Gamma_M \leq \gamma_i] = \prod_{i=1}^M P_{\Gamma_i}(\gamma_i)$$

$$p(\gamma_i) = \frac{1}{\gamma_0} e^{-\gamma_i/\gamma_0}, \quad \gamma_0 \geq 0 \quad \gamma_0 = 2\sigma^2 \frac{E_b}{N_0}$$

$$P_\Gamma(\gamma) = \Pr[\Gamma \leq \gamma] = \Pr[\max\{\Gamma_i \leq \gamma\}]$$

$$= \Pr[\Gamma_1, \Gamma_2, \dots, \Gamma_M \leq \gamma] = (1 - e^{-\gamma/\gamma_0})^M \quad P(\gamma) = 1 - e^{-\gamma/\gamma_0} \sum_{i=1}^M \frac{1}{(i-1)!} \left(\frac{\gamma}{\gamma_0} \right)^{i-1}$$

N	Harmonic	cos(mu)	Equivalent expression
1	m = 0	cos(0)	1
2	m = 1	cos(u)	cos(u)
3	m = 2	cos(2u)	2 cos^2(u) - 1
4	m = 3	cos(3u)	4 cos^3(u) - 3 cos(u)
5	m = 4	cos(4u)	8 cos^4(u) - 8 cos^2(u) + 1
6	m = 5	cos(5u)	16 cos^5(u) - 20 cos^3(u) + 5 cos(u)
7	m = 6	cos(6u)	32 cos^6(u) - 48 cos^4(u) + 18 cos^2(u) - 1
8	m = 7	cos(7u)	64 cos^7(u) - 112 cos^5(u) + 56 cos^3(u) - 7 cos(u)

$$T_m(z) = \cos(m \cos^{-1}(z)) \quad -1 \leq z \leq 1$$

$$T_m(z) = \cosh(m \cosh^{-1}(z)) \quad |z| > 1$$

$$\log_a x = y \Leftrightarrow a^y = x \quad \sin(\theta) = \frac{1}{2j} (e^{j\theta} - e^{-j\theta})$$

$$\log_a x = \frac{\log_b x}{\log_b a} \quad \cos(\theta) = \frac{1}{2} (e^{j\theta} + e^{-j\theta})$$