TLT-6306: RF Equipment for Wireless Networks
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Calculators are allowed in this exam.

Exam 30.01.2012

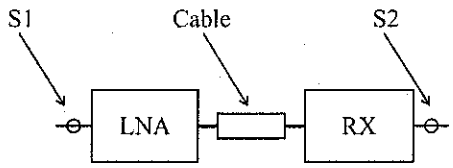
Answer to all questions!
Total maximum points: 30 (+ exercise bonus)

1. Explain shortly:

a) Receiver sensitivity **(2 p)** b) Space diversity **(2 p)** c) Phase noise **(2 p)**

2. Explain the non-idealities (the phenomena) caused by typical analogue receiver front end components.

3. Using the values given in Figure 1, calculate the signal-to-noise ratio (SNR) at the receiver output (S2), when it is measured that the SNR at the LNA input (S1) is 13 dB. **(6p)**



F_{LNA} = 2 dB, F_{RX} = 5 dB
G_{LNA} = 15 dB,
Cable loss between LNA and RX unit: 1 dB
Figure 1

4. a) What additional RF-equipment you need and how to make the connections in Figure 2? Space diversity reception is implemented and only two feeders can be used. Antenna 2 is the diversity antenna. **(3p)**

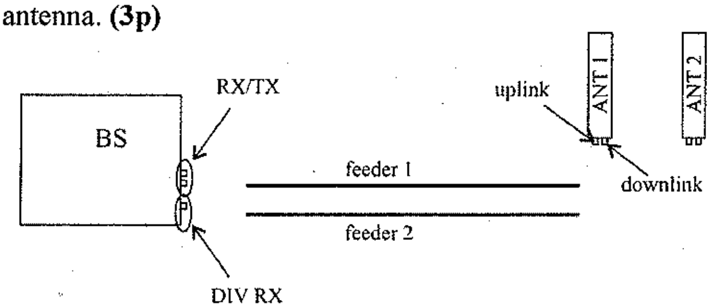


Figure 2

b)
How does the tapper differ from splitter? Consider a 2-way tapper. What is the power ratio in decibel scale between the output port #2 and the input port, if the power ratio between output port #1 and the input port is set to -7 dB. **(3p)**

5. On a *Rayleigh fading channel*, the received average signal energy is measured to be 17 dB above the noise level. Assume additive white Gaussian noise. The receiver in use is assumed to work satisfactory if the SNR is larger than 6 dB. How many branches are required for a diversity system exploiting *selection combining* to obtain a time availability of 99.5%?

Some (more or less) useful equations:

$$G_a = \frac{p_1}{p_i}, G_b = \frac{p_2}{p_1}, G_c = \frac{p_o}{p_2}$$

$$\frac{p_o}{p_i} = \frac{p_1}{p_i}\frac{p_2}{p_1}\frac{p_o}{p_2} = G_a G_b G_c$$

$$\log_{10}\frac{p_o}{p_i} = \log_{10} G_a + \log_{10} G_b + \log_{10} G_c$$

$$10\log_{10}\frac{p_o}{p_i}$$

$$10\log_{10}\frac{P_1[W]}{1mW} \qquad \frac{10^{\frac{P[dBm]}{10}}}{1000}$$

$$v_n = \sqrt{\frac{4hfBR}{e^{hf/kT}-1}}$$

$$P_n = \frac{\left(\frac{v_n}{2}\right)^2}{R} = \frac{v_n^2}{4R} = \frac{\left(\sqrt{4kTBR}\right)^2}{4R} = kTB$$

$$Y = \frac{P_1}{P_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1 \qquad 2f_1 - f_2 、 2f_2 - f_1 \qquad F = \frac{CNR_{in}}{CNR_{out}}$$

$$\lambda = \frac{c}{f} \qquad \beta = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\varepsilon}$$

$$\operatorname{div}\mathbf{D}(\mathbf{r},t) = \rho(\mathbf{r},t)$$

$$\operatorname{div}\mathbf{B}(\mathbf{r},t) = 0$$

$$\operatorname{curl}\mathbf{E}(\mathbf{r},t) = -\frac{\partial\mathbf{B}(\mathbf{r},t)}{\partial t}$$

$$\operatorname{curl}\mathbf{H}(\mathbf{r},t) = \frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t} + \mathbf{J}(\mathbf{r},t)$$

$$A_z = \iiint_{v'} \mu J_z \frac{e^{-j\beta R}}{4\pi R} dv'$$

$$A_z = \int \mu I(z') \frac{e^{-j\beta(r - z'\cos\theta)}}{4\pi r} dz' = \frac{\mu e^{-j\beta r}}{4\pi r} \int I(z') e^{j\beta z'\cos\theta} dz'$$

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots + \frac{F_n - 1}{G_1 G_2 \ldots G_{n-1}}$$

$$T_{cas} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \ldots + \frac{T_{en}}{G_1 G_2 \ldots G_{n-1}}$$

$$R = \frac{2L^2}{\lambda} \qquad R = 0.62\sqrt{L^3/\lambda}$$

0 dBd = 2.15 dBi

$$SSL_{dB} = 20\log_{10}\frac{|F(SSL)|}{|F(\max)|}$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$h = 6.546 \cdot 10^{-34}$ Jsec, Planck's constant

c = 299 792 458 *m/s*, speed of light

$k = 1.38 \cdot 10^{-23}$ J/°K, Boltsmann's constant

$$R_a = R_r + R_l \qquad Z_a = R_a + jX_a \qquad V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$\frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad -20\log_{10}|\Gamma| \text{ dB}$$

$$F(\theta,\phi) = g(\theta,\phi) \cdot f(\theta,\phi)$$

$$D = \frac{U_{\max}}{U_{ave}} \qquad \varepsilon_r = \frac{P}{P_{in}} \qquad G = \varepsilon_r D$$

$$G(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{in}} \qquad U = \frac{dP}{d\Omega} \qquad P = \frac{1}{2} I^2 R_a$$

$$I(z) = I(0)\sin\left[\beta\left(\frac{L}{2} - |z|\right)\right] \qquad AF = \sum_{n=0}^{N-1} A_n e^{jn\psi} \qquad \psi = \beta d\cos(\theta) + \alpha$$

$$N_{UL\_R} = k \cdot T \cdot B \cdot F_R \cdot G_{T\_UL}$$

$$N_{UL\_BS} = k \cdot T \cdot B \cdot F_{BS}$$

$$N_{UL} = N_{UL\_R} + N_{UL\_BS} = k \cdot T \cdot B\left(F_{BS} + G_{T\_UL} \cdot F_R\right)$$

$$AF = A_0 e^{j(N-1)\psi/2} \frac{\sin(N\psi/2)}{\sin(\psi/2)}$$

$$AF = I_0 e^{-j\xi_0} + I_1 e^{-j\xi_1} + I_2 e^{-j\xi_2} + \cdots I_M e^{-j\xi_M}$$

$$f(\psi) = \frac{\sin(N\psi/2)}{N\sin(\psi/2)} \qquad \mathbf{W} = \frac{1}{L}\mathbf{S}_0$$

$$E(\theta,\phi) = \sum_{m=1}^{M}\sum_{n=1}^{N} W_{mn} e^{j(2\pi/\lambda)(md_x\cos(\theta) + nd_y\sin(\phi)\sin(\theta))}$$

Dolph Chebyshev

Odd N	Even N
$E(\theta,\phi) = \sum_{n=0}^{\left(\frac{N-1}{2}\right)} a_n \cos(2nu)$	$E(\theta,\phi) = \sum_{n=0}^{\left(\frac{N-1}{2}\right)} a_n \cos[(2n+1)u]$

$$u = \frac{\pi d}{\lambda}\sin(\theta)\sin(\phi)$$

$$C_{SISO} = \log_2\left(1 + \frac{P}{\sigma_n^2}|h(\tau,t)|^2\right) \qquad C_{SIMO} = \log_2\left(1 + \frac{P}{\sigma_n^2}\sum_{i=1}^{N}|h_i(\tau,t)|^2\right)$$

$$C_{MISO} = \log_2\left(1 + \frac{1}{M}\frac{P}{\sigma_n^2}\sum_{j=1}^{M}|h_j(\tau,t)|^2\right) \qquad C_{MIMO} = \sum_{k=1}^{K}\log_2\left(1 + \lambda_k\frac{P_k}{\sigma_n^2}\right)$$

$$\rho_{12} = \mathrm{E}\left[\sum_{i=1}^{n_s} e^{-j\phi_i}\right] = \mathrm{E}\left[\sum_{i=1}^{n_s} e^{-jkd\sin(\theta)}\right] \qquad J_0(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{jx\cos(\alpha)} d\alpha$$

N	Harmonic	cos(u)	Equivalent expression
1	$m = 0$	$\cos(0)$	1
2	$m = 1$	$\cos(u)$	$\cos(u)$
3	$m = 2$	$\cos(2u)$	$2\cos^2(u) - 1$
4	$m = 3$	$\cos(3u)$	$4\cos^3(u) - 3\cos(u)$
5	$m = 4$	$\cos(4u)$	$8\cos^4(u) - 8\cos^2(u) + 1$
6	$m = 5$	$\cos(5u)$	$16\cos^5(u) - 20\cos^3(u) + 5\cos(u)$
7	$m = 6$	$\cos(6u)$	$32\cos^6(u) - 48\cos^4(u) + 18\cos^2(u) - 1$
8	$m = 7$	$\cos(7u)$	$64\cos^7(u) - 112\cos^5(u) + 56\cos^3(u) - 7\cos(u)$

$$\rho_{12} = \int_0^{2\pi} p(\theta) e^{-jkd\sin(\theta)} d\theta \qquad \rho(d) = \frac{1}{2\pi}\int_0^{2\pi} e^{-jkd\sin(\theta)} d\theta = \left|J_0\left(\frac{2\pi d}{\lambda}\right)\right|$$

$$P_\Gamma(\gamma_t) = \Pr[\Gamma \le \gamma_t] \qquad \Gamma = \max\{\Gamma_1, \Gamma_2, \cdots \Gamma_M\}$$

$$= \Pr[\Gamma_1, \Gamma_2, \cdots \Gamma_M \le \gamma_t] = \prod_{i=1}^{M} P_{\Gamma_i}(\gamma_t)$$

$$T_m(z) = \cos\left(m\cos^{-1}(z)\right) \qquad -1 \le z \le 1$$

$$T_m(z) = \cosh\left(m\cosh^{-1}(z)\right) \qquad |z| > 1$$

$$p(\gamma_i) = \frac{1}{\gamma_0} e^{-\gamma_i/\gamma_0}, \quad \gamma_0 \ge 0 \qquad \gamma_0 = 2\sigma^2\frac{E_b}{N_0}$$

$$\log_a x = y \Leftrightarrow a^y = x \qquad \sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

$$\log_a x = \frac{\log_b x}{\log_b a} \qquad \cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$P_\Gamma(\gamma) = \Pr[\Gamma \le \gamma] = \Pr[\max\{\Gamma_i \le \gamma\}]$$

$$= \Pr[\Gamma_1, \Gamma_2, \cdots \Gamma_M \le \gamma] = \left(1 - e^{\gamma_i/\gamma_0}\right)^M \qquad P(\gamma) = 1 - e^{-\gamma/\gamma_0}\sum_{i=1}^{M}\frac{1}{(i-1)!}\left(\frac{\gamma}{\gamma_0}\right)^{i-1}$$