

TAMPERE UNIVERSITY OF TECHNOLOGY
Department of Software Systems

OHJ-2306 Introduction to Theoretical Computer Science
prof. Tapio Elomaa

Examination
Apr. 5, 2011

Neither calculators nor any other extra material is allowed in the exam.

All students are required to answer questions 1 and 2. In addition, you may choose to answer any two questions from among 3–6. Answer only two of questions 3–6.

The maximum score for questions 1 and 2 is 8 points and for questions 3–6 7 points. In total 30 points.

Give careful and detailed answers to the questions!

ANSWER QUESTIONS 1 AND 2

1. Let w^R denote the reverse of string w ; i.e. if

$$w = a_1 a_2 \dots a_n,$$

then

$$w^R = a_n \dots a_2 a_1.$$

A string is a *palindrome* if $w = w^R$ (for example, "Step on no pets"). Let us examine the language of palindromes over the alphabet $\{a, b\}$:

$$\text{PAL} = \{ w \in \{a, b\}^* \mid w = w^R \}.$$

- (a) Show that PAL is not a regular language.
(b) Show that PAL is a context-free language.
2. Show that the halting problem of Turing machines

$$\text{HALT}_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM and halts on input } w \}$$

is undecidable. You may assume that the universal language U over the binary alphabet is not decidable. Is HALT_{TM} semi-decidable? Justify your answer.