SGN-2016 DIGITAL LINEAR FILTERING I

Final Examination 1.3.2011 by Tapio Saramäki

NO literature in the examination, short, compact, and concise answers are preferred. **The Faculty's calculator is allowed to be used. No need to return the exam questions.**

1. Explain shortly (using formulas and/or words) the meanings of the following terms: (1) Difference equation; (2) Transposed form structure; (3) Maximally flat approximation; (4) Filter stability; (5) L_∞-norm scaling; and (6) Rounding and truncation.

2. (a) Under certain conditions an FIR filter is defined to have a linear phase. What are these conditions in terms of the impulse response of the filter?

   (b) Consider an FIR filter of order 5. The impulse-response coefficients of this filter are given by $h[0] = h[1] = h[2] = h[3] = h[4] = h[5] = 1/6$. Express the frequency response of this filter in the simplest possible form. What are the phase and group delay response of this filter?

3. The traditional techniques for designing linear-phase FIR filters are based on the use of windowing and the Remez multiple exchange algorithm. What are the basic ideas behind these techniques? Compare these techniques with each other.

4. (a) How to use the bilinear transformation for designing recursive digital filters with the aid of analog filters? Why is the bilinear transformation still a good technique for generating recursive digital filters?

   (b) Design a second-order Butterworth digital filter with the aid of the bilinear transformation in such a way that the resulting squared-magnitude function achieves the value of unity at $\omega = 0$ and the value of $1/2$ ($A_p = 3.0103$ dB) at the passband edge $\omega_p = \pi/2$.. Note that there is no need to find the stopband edge.

5. Consider a recursive digital filter that is implemented as shown in the following figure. It is desired to implement the transfer function of this figure using two's complement arithmetic. The data wordlength is 1+8 bits. Determine the largest value for c for which there are no overflows in $w(n)$ (the worst-case or safety scaling). After the proper scaling, what is the variance of the output noise due to the multiplication roundoff errors? Recall that the variance of the quantization noise (for both rounding and truncation) is given by $\sigma_e^2 = 2^{-2b}/12$.