SGN-1159 Introduction to Signal Processing
Exam - 19.10.2011

Use of calculators:

D. The own programming calculator is allowed to use.

Instructions: Write your name on **every** page in CAPITAL LETTERS and your student number as well. Number pages consecutively. You have to solve six problems. The total number of points is 30 points.

1. (5 points) An L-th order moving average filter is a system that, for an input $x[n]$ produces the output:

$$y[n] = \frac{1}{1+L}\sum_{k=0}^{L} x[n-k]$$

Is this system linear? Is it time-invariant? Justify your answers. Find the system's impulse response $h[n]$ and frequency response $H(e^{j\omega})$.

2. (5 points) Consider the system defined by the difference equation

$$y[n] = ay[n-1] + bx[n] + x[n-1]$$

where a and b are real, and $|a| < 1$.

(a) Find the relationship between a and b that must exist if the frequency response is to have a constant magnitude for all ω, that is $|H(e^{j\omega})| = 1$.

(b) Assuming that this relationship is satisfied, find the output $y[n]$ of the system when $a = \frac{1}{2}$ and $x[n] = \left(\frac{1}{2}\right)^n \mu[n]$.

3. (5 points) Consider the system of Figure 1, where the input unit-energy continuous-time signal $x_a(t)$ has a band-limited spectrum $X_a(j\Omega)$, as sketched in Figure 2(a), and is being sampled at the Nyquist rate. The discrete-time processor is an ideal lowpass filter with a frequency response $H(e^{j\omega})$, as shown in Figure 2(b), and has a cutoff frequency $\omega_c = \Omega_m T_s/2$ where T_s is the sampling period. Sketch as accurately as possible the spectrum $Y_a(j\Omega)$ of the output continuous-time signal $y_a(t)$. What is the energy of the output signal $y_a(t)$?

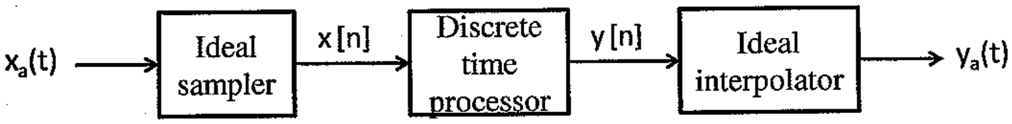

Figure 1: Problem 3. Block diagram of the system

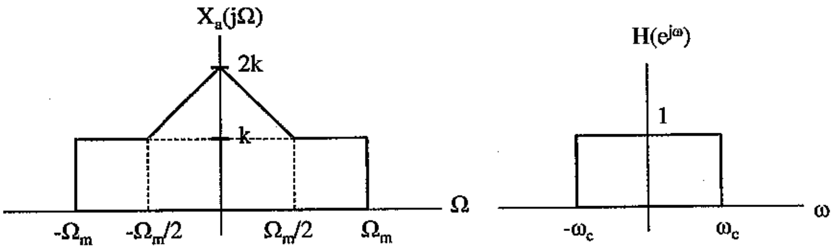

Figure 2: Problem 3. Spectrum of the input and frequency response of the digital filter. The scalar value k on the left figure is unknown.

4. (5 points) Compute the 4-point DFTs of the two real sequences:

$$x_1 = \{1,\ 4,\ -2,\ 0\}$$
$$x_2 = \{-2,\ 0,\ 1,\ 3\}$$

using ONLY a single 4-point DFT. NOTE: Computing the DFT of each sequence individually will not give you any point!

5. (5 points) Using z-transform methods, determine the explicit expression for the output $y[n]$ of each of the following causal LTI discrete-time systems, with impulse responses and inputs as indicated:

a) $h[n] = (-0.2)^n \mu[n], x[n] = (0.3)^n \mu[n]$,

b) $h[n] = (-0.7)^n \mu[n], x[n] = (-0.3)^n \mu[n]$.

6. (5 points) Find the inverse Z-transform of

$$X(z) = \frac{1 - 3z^{-5}}{(1 - 0.2z^{-1})(1 + 0.6z^{-1})} \qquad \text{ROC:} 0.2 < |z| < 0.6$$