TAMPERE UNIVERSITY OF TECHNOLOGY
Department of Software Systems

OHJ-2306 **Introduction to Theoretical Computer Science**     Examination
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*Calculators or any other extra material is **not** allowed in the exam.*

*All students are required to answer questions 1 and 2. In addition, you may choose to answer any two questions from among 3–6. The maximum score for questions 1 and 2 is 8 points and for questions 3–6 7 points. In total 30 points.*

*Give careful and detailed answers to the questions!*

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### ANSWER QUESTIONS 1 AND 2

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1. Let $AB$ be the language

    $$AB = \{\, (ab)^n \mid n \geq 0 \,\}.$$

    Give constructive answers to the following questions.

    (a) Is $AB$ a regular language?

    (b) Can language $AB$ be recognized with a pushdown automaton?

    (c) Can language $AB$ be generated by a context-free grammar?

2. Define the *universal language* $U$ over the binary alphabet.

    (a) Show that $U$ is not decidable. Take advantage of the knowledge that the "diagonal language"

    $$D = \{\, \langle M \rangle \in \{\, 0, 1 \,\}^* \mid \langle M \rangle \notin L(M) \,\}$$

    is not Turing-recognizable. Is $U$ Turing-recognizable?

    (b) Show that $U$ is TR-complete.

3. Let $A, B \subseteq \Sigma^*$ be decidable languages. Prove that then also languages

    $$\overline{A} = \Sigma^* \setminus A, A \cup B, \text{ and } A \cap B$$

    are decidable.

4. Define the Kolmogorov complexity $K(x)$ of a binary string $x$. When is $x$ $c$-sompressible? Show that incompressible strings of every length exist.

5. What do the following mean *exactly*?

    (a) $A \in P$;

    (b) $A \leq^p_m B$;

    (c) PSPACE = NPSPACE.

6. What are the mutual inclusion relations of the following language classes? Explain your answer.

    (a) Context-free languages,
    (b) Turing-recognizable languages, and
    (c) regular languages.

    Attach the following formalisms to the above-mentioned language classes: push-down automata, unrestricted grammars, deterministic Turing machines, context-free grammars, nondeterministic finite automata, regular expressions, and right-linear context-free grammars.