TLT-2716, Teletraffic theory: Queuing theory

Part 1: Theoretical questions

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Answer the following questions.

1. Give a definition of a Markov process.
2. What is the distribution of sojourn time in the state of discrete-time MC?
3. When the Markov process is said to be homogenous?
4. What is the residual lifetime?
5. Give a meaning of each position in Kendall's notation.
6. Give two examples of queuing systems in Kendall's notation.
7. Give a definition of memoryless property.
8. Which two distributions are characterized by memoryless property?
9. Which metrics does Little's result relate?
10. For which queuing systems Little result holds?
11. Formulate Kleinrock's principle?
12. For which queuing systems Kleinrock's principle holds?
13. Formulate PASTA property.
14. For which queuing systems PASTA principle holds?
15. What the rate conservation law states?

Part 2: Theoretical analysis

16. Consider M/M/1/K queue, λ, μ – arrival and service rates.

I. What is the equilibrium condition for this system? Why? Explain.
II. How the system state is defined? Why is it possible? Explain.
III. Draw the state transition diagram for this system.
IV. Get a global balance equation for state 3.
V. Get linear equations describing the system at steady-state.
VI. Derive an expression for steady-state probabilities.

Hint: it is not needed to determine the sum of series you may encounter.

VII. Derive an expression for the mean number of customers in the system.
VIII. Derive an expression for mean waiting time in the system.

17. Consider M/G/1 queuing system. Complete the following tasks.

I. Name at least two methods applicable to this system.
II. How time points of the imbedded Markov chain should be chosen? Why?

18. Consider GI/M/1 queuing system. Complete the following tasks.

I. Name the method applicable for such system that we used in lectures.
II. How time points of imbedded Markov chain should be chosen? Why?
III. Draw time diagram of interarrival time between i^{th} and $(i+1)^{th}$ arrivals. Denote and explain all events associated with this interarrival time and needed for further analysis of this queuing system.
IV. Draw state transition diagram of imbedded Markov chain.

Hint: it is not needed to derive expression for transition probabilities or steady state of the imbedded Markov chain.

Part 3: Numerical evaluation

19. **In the trunk group with m=6 trunks there are on the average 20 calls per hour. The mean holding time of a call is exponentially distributed with mean 5 min. Determine:**
    I. Mean interarrival time;
    II. Mean service rate;
    III. Offered traffic load to the system.
    IV. Server utilization.
20. **For a computing system with one processor the processing time per customer is exponentially distributed with an average time of 6 minutes. Customers arrive according to Poisson process at an average rate of one customer every 8 minutes and are processed on a FCFS basis. Determine:**
    I. Kendall's notation of the queuing system;
    II. Mean number of customers in the system;

Hint: the mean waiting time in the system: $E[W] = \dfrac{1}{\mu(1-\rho)}$, ρ *is the offered traffic load,* μ *is the service rate.*

    III. Probability that an arriving customer require less or equal to 20 minutes to leave the system after successful service.

Hint: distribution of sojourn time in the system is: $W(t) = \Pr\{w \le t\} = 1 - e^{-\mu(1-\rho)t}$.

21. **Consider M/M/m/m queuing system. Assume that the mean arrival rate is 180 customers per hour, mean service time is 0.2 minute and m=3. Determine the following:**
    I. Is this system syable? Why?
    II. Mean interarrival time;
    III. Mean service rate;
    IV. How to get the effective arrival rate? Is it different compared to the arrival rate?

Hint: in the last question no computation are needed just step-by-step explanation.