

MAT-1035X Insinöörimatematiikka 5 / viihjeitä

$$1. \iint_{R_{xy}} f(x, y) dx dy = \iint_{R_{uv}} f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du dv$$

$$2. \frac{\partial(x, y)}{\partial(u, v)} \cdot \frac{\partial(u, v)}{\partial(x, y)} = 1$$

$$3. m = \iint_R \rho(x, y) da, \quad J = \iint_R d(x, y)^2 \rho(x, y) da$$

$$x_0 = \frac{1}{m} \iint_R x \rho(x, y) da, \quad y_0 = \frac{1}{m} \iint_R y \rho(x, y) da$$

$$4. \begin{cases} x = \rho \sin \phi \cos \theta \\ y = \rho \sin \phi \sin \theta \\ z = \rho \cos \phi \end{cases} \Rightarrow \frac{\partial(x, y, z)}{\partial(\rho, \phi, \theta)} = \rho^2 \sin \phi$$

$$5. \frac{dy}{dx} + a(x)y = f(x); \quad y = e^{-A(x)} \left(\int f(x) e^{A(x)} dx + C \right), \quad A'(x) = a(x)$$

$$6. y(x) = c_1(x)y_1(x) + c_2(x)y_2(x)$$

$$\begin{cases} c_1'(x)y_1(x) + c_2'(x)y_2(x) = 0 \\ c_1'(x)y_1'(x) + c_2'(x)y_2'(x) = f(x) \end{cases}$$

$$7. a \cos \omega t + b \sin \omega t = A \sin(\omega t + \phi)$$

$$A = \sqrt{a^2 + b^2} \quad \text{ja} \quad \cos \phi = \frac{b}{A}, \quad \sin \phi = \frac{a}{A} \quad \text{eli} \quad \phi = \arctan \frac{a}{b} \quad (\pm \pi)$$

$$8. f(x) = c e^{\alpha x}$$

$$y(x) = K e^{\alpha x} \quad \text{jos } \alpha \text{ ei ole kar. yhtälön juuri}$$

$$y(x) = K x e^{\alpha x} \quad \text{jos } \alpha \text{ on kar. yhtälön 1-kertainen juuri}$$

$$y(x) = K x^2 e^{\alpha x} \quad \text{jos } \alpha \text{ on kar. yhtälön 2-kertainen juuri}$$

$$9. y'' + \omega^2 y = p \cos \omega x + q \sin \omega x$$

$$y(x) = A x \cos \omega x + B x \sin \omega x, \quad A = -\frac{q}{2\omega} \quad \text{ja} \quad B = \frac{p}{2\omega}$$

$$10. y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 y' + a_0 y = 0$$

(i) yksinkertainen reaali juuri λ_1 ; ratkaisu $e^{\lambda_1 x}$

(ii) yksinkertainen imaginaarijuuri $\alpha \pm j\beta$; ratkaisu $e^{\alpha x} \cos \beta x$ ja $e^{\alpha x} \sin \beta x$

(iii) k-kertainen reaali juuri λ_1 , ratkaisu

$$e^{\lambda_1 x}, x e^{\lambda_1 x}, x^2 e^{\lambda_1 x}, \dots, x^{k-1} e^{\lambda_1 x}$$

(iv) k-kertainen imaginaarijuuri $\alpha \pm j\beta$, ratkaisu

$$e^{\alpha x} \cos \beta x, x e^{\alpha x} \cos \beta x, \dots, x^{k-1} e^{\alpha x} \cos \beta x$$

$$e^{\alpha x} \sin \beta x, x e^{\alpha x} \sin \beta x, \dots, x^{k-1} e^{\alpha x} \sin \beta x$$

$$11. \mathbf{x}' = A \mathbf{x} + \mathbf{b}(t) \dots \mathbf{x}(t) = X(t) \mathbf{c} + \mathbf{x}_p(t)$$

$$X(t) = \left[\mathbf{v}_1 e^{\lambda_1 t}, \mathbf{v}_2 e^{\lambda_2 t}, \dots, \mathbf{v}_n e^{\lambda_n t} \right]$$

$$\lambda_{1,2} = \alpha \pm j\beta, \quad \mathbf{w}_{1,2} = \mathbf{u} \pm j\mathbf{v}, \dots, \quad \text{Re}(\mathbf{w}_1 e^{\lambda_1 t}), \quad \text{Im}(\mathbf{w}_1 e^{\lambda_1 t})$$

$$12. \mathbf{x}' = A \mathbf{x} + e^{\lambda t} \mathbf{k} \dots \mathbf{x}(t) = e^{\lambda t} \mathbf{v} \dots (A - \lambda I) \mathbf{v} = -\mathbf{k}$$

13. Integrointikaavoja:

$$\int f(g(t))g'(t) dt = F(g(t)); \quad F' = f, \quad \int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$$

$$\int u(x)v'(x) dx = u(x)v(x) - \int v(x)u'(x) dx$$