Write your name and student number to each separate answer sheet. Neither calculators nor any other extra material is allowed in the exam.

All students are required to answer questions 1 and 2. In addition, you may choose to answer any two questions from among 3–6. The maximum score for questions 1 and 2 is 8 points and for questions 3–6 7 points. In total 30 points.

Give careful and detailed answers to the questions!

ANSWER QUESTIONS 1 AND 2

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1. Let AB be the language

$$AB = \{\, a^n b^n \mid n \geq 0 \,\}.$$

Give well-grounded answers to the following questions.

(a) Is AB a regular language?

(b) Can language AB be recognized with a pushdown automaton?

(c) Can language AB be generated by a context-free grammar?

2. Define the *universal language* U over the binary alphabet. Show that U is not decidable. Take advantage of the knowledge that the "diagonal language"

$$D = \{\, \langle M \rangle \in \{\, 0, 1 \,\}^* \mid \langle M \rangle \notin L(M) \,\}$$

is not Turing-recognizable. Is U Turing-recognizable?

3. Let $A, B \subseteq \Sigma^*$ be decidable languages. Prove that then also languages

$$\overline{A} = \Sigma^* \setminus A, A \cup B, \text{ and } A \cap B$$

are decidable.

4. Show that incompressible strings of every length exist.

5. Two numbers are *relatively prime* if 1 is the largest integer that evenly divides them both. For instance, 10 and 21 are relatively prime. Is recognizing pairs of numbers that are relatively prime a problem that can be solved efficiently? Justify your answer carefully.

6. Prove that if B is NP-complete and $B \leq^p_m C$ for some $C \in$ NP, then also C is NP-complete.